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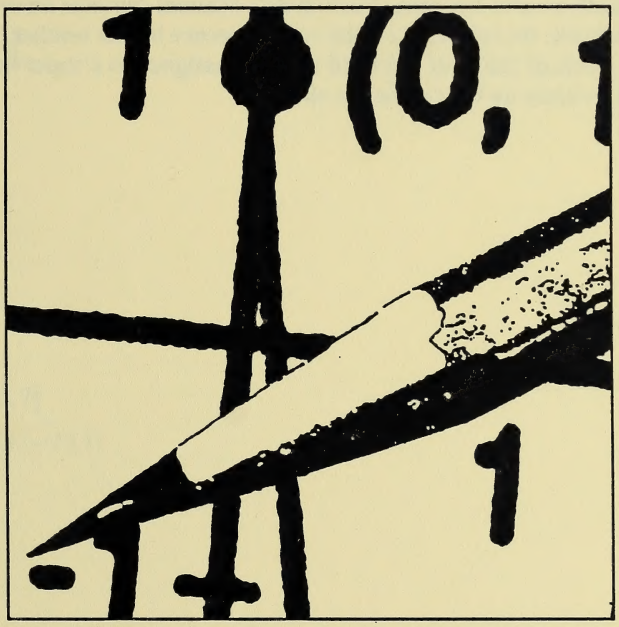
MATHEMATICS 31

LEARNING FACILITATOR'S MANUAL



UNIT 7: INNER PRODUCT

Distance Learning

Alberta EDUCATION

Note

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Mathematics 31
Learning Facilitator's Manual
Unit 7
Inner Product
Alberta Distance Learning Centre
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Topic 1: Definition and Evaluation

1. Find $\vec{u} \bullet \vec{v}$ for the following.

(2) a. $\vec{u} = [-2, 8]$ and $\vec{v} = [7, 3]$

$$\begin{aligned}\vec{u} \bullet \vec{v} &= [-2, 8] \bullet [7, 3] \\ &= (-2)(7) + (8)(3) \\ &= -14 + 24 \\ &= 10\end{aligned}$$

(2) b. $\vec{u} = [3, 1, -2]$ and $\vec{v} = [4, 0, 7]$

$$\begin{aligned}\vec{u} \bullet \vec{v} &= [3, 1, -2] \bullet [4, 0, 7] \\ &= (3)(4) + (1)(0) + (-2)(7) \\ &= 12 + 0 - 14 \\ &= -2\end{aligned}$$

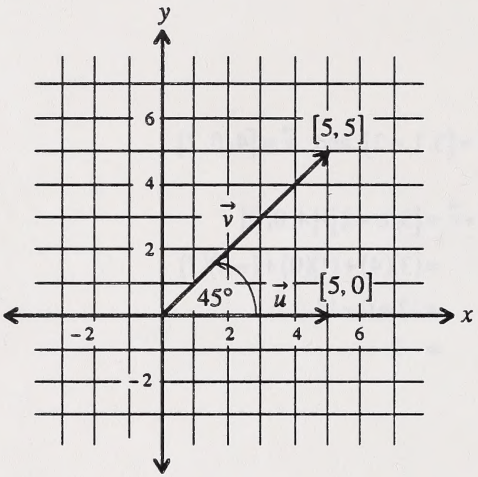
(4) 2. Find x if $[3, x, -2] \bullet [2x, -1, 6] = 8$.

$$\begin{aligned}(3)(2x) + (x)(-1) + (-2)(6) &= 8 \\ 6x - x - 12 &= 8 \\ 5x - 12 &= 8 \\ 5x &= 20 \\ x &= 4\end{aligned}$$

(4) 3. Find $\vec{u} \bullet 2\vec{v} + \vec{v} \bullet \vec{w}$ given $\vec{u} = [6, 4]$, $\vec{v} = [-1, -3]$, and $\vec{w} = [2, -5]$.

$$[6,4] \bullet 2[-1,-3] + [-1,-3] \bullet [2,-5] = -12 - 24 - 2 + 15$$
$$= -23$$

(8) 4. Verify that $\vec{u} \bullet \vec{v} = \left|\vec{u}\right|\left|\vec{v}\right| \cos\theta$ if $\vec{u} = [5, 0]$ and $\vec{v} = [5, 5]$. Draw a graph to determine the angle θ between $\vec{u}$ and $\vec{v}$.



By observation $\theta = 45°$.

$$\left|\vec{u}\right| = \sqrt{5^2 + 0^2}$$
$$= 5$$
$$\left|\vec{v}\right| = \sqrt{5^2 + 5^2}$$
$$= \sqrt{50}$$

LS	RS
$\vec{u} \bullet \vec{v}$	$\left\|\vec{u}\right\|\left\|\vec{v}\right\| \cos 45°$
$[5, 0] \bullet [5, 5]$	$(5)\left(\sqrt{50}\right) \cos 45°$
$(5)(5) + (0)(5)$	$5\sqrt{50}\left(\frac{1}{\sqrt{2}}\right)$
25	25

LS = RS

Topic 1

_____ marks

Topic 2: Projections and Work

1. Find the projection of vector $\vec{w}$ on the vector $\vec{v}$ for the following.

(4) a. $\vec{w}=[-3,7]$ and $\vec{v}=[5,1]$

$$\frac{\vec{w}\bullet\vec{v}}{\left|\vec{v}\right|}=\frac{[-3,7]\bullet[5,1]}{\sqrt{25+1}}$$

$$=\frac{-15+7}{\sqrt{26}}$$

$$=\frac{-8}{\sqrt{26}}$$

$$=\frac{-8\sqrt{26}}{26}$$

$$=\frac{-4\sqrt{26}}{13}$$

(4) b. $\vec{w}=[5,2,4]$ and $\vec{v}=[-3,1,2]$

$$\frac{\vec{w}\bullet\vec{v}}{\left|\vec{v}\right|}=\frac{[5,2,4]\bullet[-3,1,2]}{\sqrt{(-3)^2+1^2+2^2}}$$

$$=\frac{5(-3)+2(1)+4(2)}{\sqrt{9+1+4}}$$

$$=\frac{-5}{\sqrt{14}}$$

$$=\frac{-5\sqrt{14}}{14}$$

(6) 2. Four points in three-space are $A(3,1,0)$, $B(5,2,3)$, $C(-1,0,2)$, and $D(-2,-1,-3)$. Find the projection of $\overrightarrow{AB}$ on $\overrightarrow{CD}$.

$\overrightarrow{AB} = [5-3, 2-1, 3-0] = [2,1,3]$

$\overrightarrow{CD} = [-2+1-0, -1, -3-2] = [-1,-1,-5]$

The projection of $\overrightarrow{AB}$ on $\overrightarrow{CD}$ is $\dfrac{\overrightarrow{AB} \cdot \overrightarrow{CD}}{\left|\overrightarrow{CD}\right|}$.

$$\frac{\overrightarrow{AB} \cdot \overrightarrow{CD}}{\left|\overrightarrow{CD}\right|} = \frac{[2,1,3] \cdot [-1,-1,-5]}{\sqrt{1+1+25}}$$

$$= \frac{-2-1-15}{\sqrt{27}}$$

$$= \frac{-18}{\sqrt{27}}$$

$$= \frac{-6}{\sqrt{3}}$$

$$= -2\sqrt{3}$$

(3) 3. A 30 N force moves an object from point $A(6,2,5)$ to point $B(4,-3,2)$. The distance the object moves is in metres, and the force is acting at an angle of 40° to $\overrightarrow{AB}$. Determine the work done to one decimal place.

$\overrightarrow{AB} = [4-6, -3-2, 2-5]$

$= [-2,-5,-3]$

$\left|\overrightarrow{AB}\right| = \sqrt{(-2)^2 + (-5)^2 + (-3)^2}$

$= \sqrt{38}$

$\text{Work} = 30\left|\overrightarrow{AB}\right| \cos 40°$

$\doteq 30\left(\sqrt{38}\right)(0.7660)$

$\doteq 141.7$

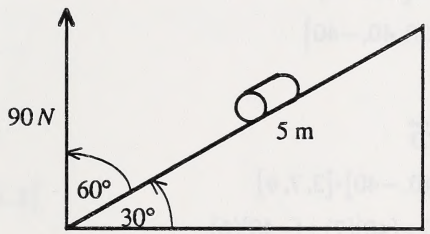
The work done is approximately 141.7 J.

(2) 4. The vector $\vec{F} = [4, 7]$ represents a force in newtons. This force moves an object along the same direction and distance (in metres) as the vector $\vec{V} = [1, 4]$. Find the work done.

$$\begin{aligned}\text{Work} &= \vec{F} \bullet \vec{V} \\ &= [4, 7] \bullet [1, 4] \\ &= 4 + 28 \\ &= 32\end{aligned}$$

The work done is 32 J.

(4) 5. How much work is done by a worker who rolls a 90 N drum up a 5 m inclined plane that is 30° to the horizontal?



$$\begin{aligned}\text{Work} &= \left|\vec{u}\right|\left|\vec{v}\right| \cos\theta \\ &= (90)(5)\cos 60° \\ &= (450)\left(\frac{1}{2}\right) \\ &= 225\end{aligned}$$

The work done is 225 J.

(6)

6. The initial point $A(1, 3, 1)$ and terminal point $B(2, 5, -1)$ determine vector $\overrightarrow{AB}$. A force of 60 N is applied to an object in a direction parallel to $\overrightarrow{AB}$. The object moves a distance (in metres) on segment CD from $C(2, 1, 1)$ to $D(5, 8, 7)$. Find the work done.

$$\overrightarrow{AB} = [2-1, 5-3, -1-1]$$
$$= [1, 2, -2]$$

$$\overrightarrow{CD} = [5-2, 8-1, 7-1]$$
$$= [3, 7, 6]$$

$$\text{Force} = \vec{F} = k[1, 2, -2]$$
$$60 = k\sqrt{1^2 + 2^2 + (-2)^2}$$
$$60 = k\sqrt{9}$$
$$60 = 3k$$
$$k = 20$$
$$\therefore \vec{F} = 20[1, 2, -2]$$
$$= [20, 40, -40]$$

$$\text{Work} = \vec{F} \cdot \overrightarrow{CD}$$
$$= [20, 40, -40] \cdot [3, 7, 6]$$
$$= 20(3) + (40)(7) + (-40)(6)$$
$$= 60 + 280 - 240$$
$$= 100$$

The work done is 100 J.

⑥ 7. A force of 20 N is applied to an object in the direction of vector $\overrightarrow{AB}$ with $A(-2,4)$ and $B(3,1)$. The object moves from point $C(-5,1)$ to point $D(0,3)$. If the distance is in metres, find the work done.

$$\overrightarrow{AB} = [3+2, 1-4]$$
$$= [5, -3]$$

$$\overrightarrow{CD} = [0-(-5), 3-1]$$
$$= [5, 2]$$

$$\vec{F} = k[5, -3]$$
$$20 = k\sqrt{25+9}$$
$$k = \frac{20}{\sqrt{34}}$$
$$\therefore \vec{F} = \frac{20}{\sqrt{34}}[5, -3]$$
$$= \left[\frac{100}{\sqrt{34}}, \frac{-60}{\sqrt{34}}\right]$$

$$\text{Work} = \vec{F} \cdot \overrightarrow{CD}$$
$$= \left[\frac{100}{\sqrt{34}}, \frac{-60}{\sqrt{34}}\right] \cdot [5, 2]$$
$$= \frac{500}{\sqrt{34}} - \frac{120}{\sqrt{34}}$$
$$= \frac{380}{\sqrt{34}}$$
$$\doteq 65.2$$

The work done is approximately 65.2 J.

Topic 2

_____ marks

Topic 3: Vector Angles and Resolution of a Vector into Two Perpendicular Components

(5) 1. Vectors $\vec{u}$ and $\vec{v}$ are perpendicular to each other. Find k if $\vec{u}=[2,3,-4]$ and $\vec{v}=[1,0,k]$.

$$\vec{u}\cdot\vec{v}=(2)(1)+(3)(0)+(-4)(k)$$
$$=2+0-4k$$

$$\cos\theta=0=\frac{\vec{u}\cdot\vec{v}}{\left|\vec{u}\right|\left|\vec{v}\right|}$$
$$0=\frac{2+0-4k}{\left|\vec{u}\right|\left|\vec{v}\right|}$$
$$0=2-4k$$
$$4k=2$$
$$k=\frac{1}{2}$$

2. Find the angle between the two vectors.

(5) a. $[3,2]$ and $[-1,5]$

Let $\vec{u}=[3,2]$ and $\vec{v}=[-1,5]$.

$$\cos\theta=\frac{\vec{u}\cdot\vec{v}}{\left|\vec{u}\right|\left|\vec{v}\right|}$$
$$=\frac{[3,2]\cdot[-1,5]}{\sqrt{3^2+2^2}\sqrt{(-1)^2+5^2}}$$
$$=\frac{7}{\sqrt{13}\sqrt{26}}$$
$$=\frac{7}{13\sqrt{2}}$$
$$=\frac{7\sqrt{2}}{26}$$
$$\doteq 0.3807$$
$$\therefore\theta\doteq 67.62^\circ$$

(5)

b. $[1,3,2]$ and $[-2,0,4]$

$$\cos\theta = \frac{[1,3,2]\bullet[-2,0,4]}{\sqrt{1^2+3^2+2^2}\sqrt{(-2)^2+0^2+4^2}}$$

$$= \frac{-2+0+8}{\sqrt{14}\sqrt{20}}$$

$$= \frac{6}{\sqrt{14}\sqrt{20}}$$

$$\doteq \frac{6}{(3.7417)(4.4721)}$$

$$\doteq 0.3586$$

$$\therefore \theta \doteq 69°$$

(8)

3. The vertices of ΔABC in three-space are $A(7,9,10)$, $B(3,11,4)$, and $C(4,7,9)$. Is ΔABC a right triangle? Prove your answer.

$$\overrightarrow{CA} = [7-4, 9-7, 10-9]$$
$$= [3,2,1]$$

$$\overrightarrow{CB} = [3-4, 11-7, 4-9]$$
$$= [-1,4,-5]$$

$$\overrightarrow{CA} \bullet \overrightarrow{CB} = [3,2,1]\bullet[-1,4,-5]$$
$$= -3+8-5$$
$$= 0$$

$\overrightarrow{CA}$ and $\overrightarrow{CB}$ are orthogonal; therefore, $\angle C = 90°$ and ΔABC is a right triangle.

(3) 4. The vector $\left[3, \frac{1}{2}\right]$ is resolved into two rectangular components $\vec{x}$ and $\vec{y}$ parallel to the x- and y-axes respectively. Find $\vec{x}$ and $\vec{y}$.

$\vec{x} = [3,0] = 3\vec{e}_1$, where $\vec{e}_1 = [1,0]$.

$\vec{y} = \left[0, \frac{1}{2}\right] = \frac{1}{2}\vec{e}_2$, where $\vec{e}_2 = [0,1]$.

(5) 5. The vector $[5,4]$ is resolved into two rectangular components $\vec{w}_1$ and $\vec{w}_2$. Find $\left|\vec{w}_1\right|$ and $\left|\vec{w}_2\right|$ if $\left|\vec{w}_1\right| = \frac{1}{3}\left|\vec{w}_2\right|$.

$$\left|\vec{w}_1\right|^2 + \left|\vec{w}_2\right|^2 = \left(\sqrt{5^2+4^2}\right)^2$$

$$\left|\vec{w}_1\right|^2 + \left|\vec{w}_2\right|^2 = 5^2 + 4^2$$

$$\left|\vec{w}_1\right|^2 + \left(3\left|\vec{w}_1\right|\right)^2 = 41$$

$$10\left|\vec{w}_1\right|^2 = 41$$

$$\left|\vec{w}_1\right|^2 = 4.1$$

$$\left|\vec{w}_1\right| = \sqrt{4.1}$$

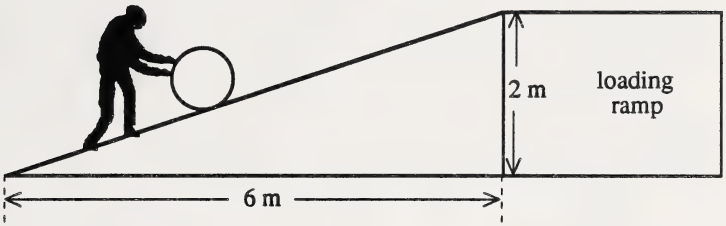
$$\doteq 2.025$$

$$\left|\vec{w}_2\right| = 3\left|\vec{w}_1\right|$$

$$\left|\vec{w}_2\right| = 3\sqrt{4.1}$$

$$\doteq 6.075$$

(5) 6. **A worker rolls a drum of gasoline up an inclined plane onto a loading ramp as shown in the diagram.**



If the force exerted by the worker (in newtons) is represented by the vector $[3, 7]$, find the component which rolls the drum up the inclined plane.

$\vec{F} = [3, 7]$

$\vec{v}(\text{incline}) = [6, 2]$

$$\text{Projection} = \frac{[3, 7] \bullet [6, 2]}{\sqrt{6^2 + 2^2}}$$

$$= \frac{18 + 14}{\sqrt{40}}$$

$$= \frac{32}{\sqrt{40}}$$

$$\doteq 5.06$$

The component which rolls the drum up the inclined plane is approximately 5.06 N.

⑨ 7. The two points $P(1,3,2)$ and $Q(5,3,6)$ determine the vector $\overrightarrow{PQ}$. $\overrightarrow{PQ}$ represents the direction of a 50 N force exerted on an object. Find the magnitude of the force parallel to the x-, y-, and z-axes.

$$\overrightarrow{PQ} = [5-1, 3-3, 6-2]$$
$$= [4, 0, 4]$$

$$\left|\overrightarrow{PQ}\right| = \sqrt{16+0+16}$$
$$= \sqrt{32}$$

$$\vec{F} = k[4, 0, 4]$$

$$\left|\vec{F}\right| = k\sqrt{32} = 50$$

$$k = \frac{50}{\sqrt{32}}$$

$$\vec{F} = \frac{50}{\sqrt{32}}[4, 0, 4]$$

$$\left|\vec{F}_x\right| = \frac{200}{\sqrt{32}}$$
$$\doteq 35.36$$

$$\left|\vec{F}_y\right| = 0$$

$$\left|\vec{F}_z\right| = \frac{200}{\sqrt{32}}$$
$$\doteq 35.36$$

The magnitude of the force parallel to the x-axis is 35.36 N, the magnitude of the force parallel to the y-axis is 0, and the magnitude of the force parallel to the z-axis is 35.36 N.

Topic 3

_____ marks

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Mathematics 31

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